Pearson Edexcel Level 3 GCE (Style)

MATHEMATICS

Advanced – Paper 2: Pure Mathematics

PRACTICE PAPER 2 (9MA0/01 STYLE)

Time: 2 hours

Total marks: 100

Instructions

- Answer all questions.
- Show sufficient working to make your methods clear.
- Give answers to three significant figures unless otherwise stated.
- You may use a calculator.

Question 1

$$y = 2x^4 - 5x^3 + \frac{6}{\sqrt{x}}, \qquad x > 0.$$

(a) Find $\frac{dy}{dx}$, giving each term in its simplest form.

(b) Find $\frac{d^2y}{dx^2}$.

(Total for Question 1 is 4 marks)

Question 2

An arithmetic sequence has first term 27 and common difference -4.

(a) Find the 10th term of the sequence.

(b) Show that the sum of the first n terms is given by

$$S_n = 29n - 2n^2.$$

(c) Given that $S_n = -330$, find the value of n.

(Total for Question 2 is 5 marks)

Question 3

The point $P(-2, 5)$ lies on the curve with equation $y = f(x)$.
Find the coordinates of the point to which P is mapped when the curve $y = f(x)$ is transformed to the curve with equation

(i) $y = f(x + 2) - 3$

(ii) $y = \frac{1}{2} f(-x)$

(iii) $y = f(2x) + 1$

(Total for Question 3 is 4 marks)

Question 4

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

(a) Express

$$\frac{11x-7}{(x-1)(2x+3)}$$

in partial fractions.

(b) Hence find

$$\int \frac{11x-7}{(x-1)(2x+3)}\,dx,$$

giving your answer as a single logarithm plus a constant.

(Total for Question 4 is 6 marks)

Question 5

Given that θ is small and measured in radians, use the small angle approximations for sin, cos and tan to show that

$$\frac{\sin 3\theta + \tan 2\theta}{1 - \cos 4\theta} \approx \frac{a}{\theta}$$

where a is a constant to be found.

(Total for Question 5 is 5 marks)

Question 6

A curve has equation

$$y = x^3 - 6x^2 + 9x + 2.$$

(a) Find the coordinates of the two stationary points of the curve.

(b) Determine the nature of each stationary point.

(c) Find the coordinates of the point of inflection of the curve.

(Total for Question 6 is 8 marks)

Question 7

Relative to a fixed origin O, the points A, B and C have position vectors

$$\overrightarrow{OA} = 3\mathbf{i} - \mathbf{j} + 2\mathbf{k}, \qquad \overrightarrow{OB} = 5\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}, \qquad \overrightarrow{OC} = \mathbf{i} - 5\mathbf{j} + 8\mathbf{k}.$$

(a) Show that A, B and C are collinear, and find the ratio $AB : BC$.

(b) The point D is such that $\overrightarrow{AD} = 2\,\overrightarrow{AB}$. Find the position vector of D and the exact distance OD.

(Total for Question 7 is 7 marks)

Question 8

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

(a) Prove that

$$\frac{1-\cos 2\theta}{\sin 2\theta} \equiv \tan\theta, \qquad \theta \neq \frac{n\pi}{2},\ n \in \mathbb{Z}.$$

(b) Hence solve, for $0 \leq x < 360°$, the equation

$$\frac{1-\cos 4x}{\sin 4x} = \sqrt{3},$$

giving your answers in degrees.

(Total for Question 8 is 8 marks)

Question 9

The value, £V, of a rare painting t years after it was purchased is modelled by

$$V = Ab^t,$$

where A and b are constants.
A graph of $\log_{10} V$ against t gives a straight line with gradient 0.018 and intercept 4.7 on the $\log_{10} V$ axis.

(a) Find the value of A and the value of b, giving each to an appropriate degree of accuracy.

(b) Interpret, in the context of the model, the value of b.

(c) Use the model to estimate the value of the painting 15 years after purchase.

(d) Find, to the nearest year, how long after purchase the model predicts the painting's value first exceeds double its purchase value.

(Total for Question 9 is 9 marks)

Question 10

A curve C has parametric equations

$$x = t^2 - 1, \qquad y = t^3 - 3t, \qquad t \in \mathbb{R}.$$

(a) Find $\dfrac{dy}{dx}$ in terms of t.

(b) Find the coordinates of the points on C where the tangent is parallel to the x-axis.

(c) Show that the curve crosses itself at the point $(2, 0)$, and find the two values of t at this point.

(Total for Question 10 is 8 marks)

Question 11

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.
The finite region R is bounded by the curve with equation $y = (4x + 2)e^{-x}$, the x-axis, the y-axis and the line $x = 2$.
Find the exact area of R, giving your answer in the form $a + be^{-2}$, where a and b are integers to be found.

(Total for Question 11 is 7 marks)

Question 12

A population of bacteria is modelled by the differential equation

$$\frac{dP}{dt} = \frac{1}{20}P(40 - P),$$

where P (in thousands) is the number of bacteria and t is the time in hours. Initially $P = 10$.

(a) Express

$$\frac{1}{P(40 - P)}$$

in partial fractions.

(b) Solve the differential equation, showing that

$$P = \frac{40}{1 + 3e^{-2t}}.$$

(c) State the long-term value of P predicted by the model, justifying your answer.

(Total for Question 12 is 9 marks)

Question 13

The equation

$$x^3 - 12x + 7 = 0$$

has a root α in the interval $[3, 4]$.

(a) Show that the equation can be rearranged into the form

$$x = \sqrt[3]{12x - 7}.$$

(b) Using the iteration $x_{n+1} = \sqrt[3]{12x_n - 7}$ with $x_1 = 3$, find x_2, x_3 and x_4, giving each to 4 decimal places.

(c) By choosing a suitable interval, show that $\alpha = 3.1240$ correct to 4 decimal places.

(Total for Question 13 is 7 marks)

Question 14

The circle C_1 has equation

$$x^2 + y^2 - 8x + 2y + 8 = 0.$$

(a) Find the coordinates of the centre and the radius of C_1.

A second circle C_2 has centre $(1, 3)$ and radius r, where r is a constant.

(b) Given that C_1 and C_2 touch externally, find the value of r.

(c) Given instead that C_1 and C_2 intersect at two distinct points, find the range of values of r, writing your answer in set notation.

(Total for Question 14 is 8 marks)

Question 15

The curve C has equation

$$x^3 + 2xy - y^2 = 12.$$

(a) Find an expression for $\dfrac{dy}{dx}$ in terms of x and y.

(b) Hence find the coordinates of the point on C, in the first quadrant, at which the tangent is parallel to the y-axis.

(Total for Question 15 is 5 marks)

END OF PAPER

TOTAL FOR PAPER: 100 MARKS